Assignment 1

Due Friday Feb 2nd, 2018

Submission Instructions: Submit solutions in a single PDF via OWL. Assignments are due at 11:59:59 pm (Eastern Time) on the date listed above. As per the course late policy, assignments submitted more than 48 hours late will **not be accepted** and a mark of zero (0) will be recorded. Email submissions will not be accepted.

1. [6 marks] Describe the language

The formal description of a DFA M is $(S = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b, c\}, \delta, start = q_0, F = \{q_3\})$, where δ is given by the following table:

	a	b	c
q_0	q_1	q_2	q_0
q_1	q_1	q_3	q_1
q_2	q_3	q_2	q_2
q_3	q_3	q_3	q_3

(a) [4 marks] Give the state diagram of this machine.

(b) [2 marks] Using *set builder* notation, describe the language accepted by M.

2. [8 marks] Identify the regular languages

For each of the following languages state whether it is regular or not. If L_i is regular, prove it by drawing a DFA or NFA (your choice) that recognizes it. If the language is not regular, give an argument (in plain English) why there is no DFA or NFA that can recognize it:

(a) [2 marks] $L_b = \{w \in \{0, 1\}^* : w$ represented as a binary integer is a power of 2$\}$

(b) [2 marks] $L_a = \{w(a^n)w : w \in \{a, b\}, n > 0\}$

(c) [2 marks] $L_c = \{wax : w, x \in \{a, b\}^*, |w|, |x| > 0\}$

(d) [2 marks] $L_d = \{wwx : w, x \in \{a, b\}^*, |w|, |x| > 0\}$

3. [10 marks] Recognizing decimal integers divisible by 3

Let string $s \in \{0-9\}^*$. Let n be string s interpreted as a decimal integer. Draw a DFA that accepts s if and only if:

$$n \equiv 0 \mod 3.$$

Assume $\varepsilon \not\equiv 0 \mod 3$.

4. [6 marks] Design NFAs

Let $\Sigma = \{a, b, c\}$. Draw an NFA recognizing each of the following languages:

(a) [2 marks] The set of strings that contain a single c,

(b) [2 marks] The set of strings that contain only 2 of the 3 possible characters (e.g., `aabba`, `bcbb` but not `aabc`),

(c) [2 marks] The set of strings that contain no consecutive digits (i.e., a 0 cannot follow a 0, etc).

5. [10 marks] An NFA in an Economy of States

Let $s \in \Sigma = \{a\}^*$. Let $|s|$ denote the length of string s. Construct a finite automaton in less than a *dozen* states that recognizes language:

$$L = \{s : \text{gcd}(|s|, 500) \neq 1\},$$

where $\text{gcd}(x, y)$ denotes the greatest common divisor between two numbers x, y.

6. [10 marks] Prove Finite Languages are Regular

We say a language L is *finite* if L contains a finite number of strings. Using induction, prove all finite languages are regular.