

SE 4472a

Information Security

Week 5:

Hash Functions



Fingerprints: An example

This file:

- ubuntu-14.04-desktop-amd64.iso

Has this SHA-1 **hash**:

- d4d44272ee5f5bf887a9c85ad09ae957bc55f89d

We've taken a 1GB file, and created a 20 byte **fingerprint**

<http://old-releases.ubuntu.com/releases/trusty/SHA1SUMS>

Hash Functions

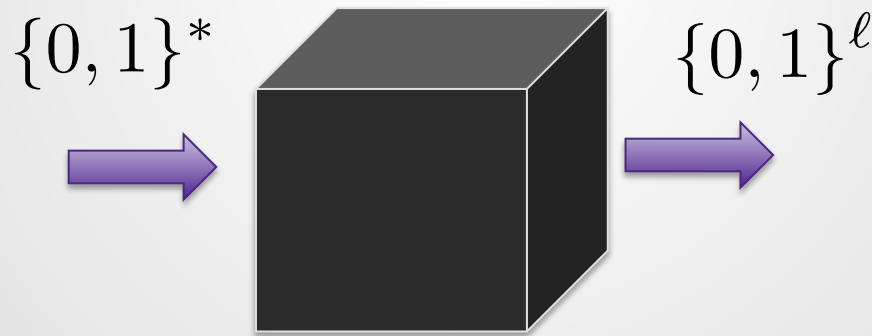
- What does it do?
 - Hash is a breakfast of chopped up meat, potatoes and vegetables
 - A **cryptographic hash** does something analogous: chops up and rearranges a string into a new string
- Why do I need it?
 - Whenever it would be beneficial to create a short fixed-length string as kind of **fingerprint**, or **digest** of a long, arbitrary-length string.

Applications Hash Function

- Hash functions in TLS
 - An **efficiency** aid for digital signatures
 - Message authentication codes (MACs)
 - Key derivation function (to turn one key into a bunch of keys)
- Other uses
 - Fingerprinting e.g., for viruses/malware, intrusion detection, chain-of-custody
 - Integrity checks, e.g. in open source code/binary distribution
 - Secure **password** storage

Ideal Cryptographic Hashes

- A pseudo-random function that accepts an **arbitrary-length** string, and produces a **fixed-length** string



- Some additional properties required

Random Oracle Model

A **random oracle** is a way to conceptualize an ideal hash function

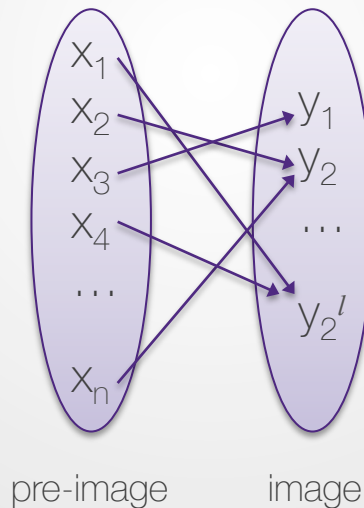
- For every arbitrary input, a **random** oracle outputs a random fixed-length string
- Each unique query is **independent** of the others
- If you repeat a query to the random oracle, it gives you the **same** answer

Random Oracles Don't Exist!

- Since there are infinitely many possible input strings, a random oracle would require **infinite memory** to maintain input-output pairs
- You could tweak the def'n and restrict the permissible input length (some real hash functions do this). This would still require **exponential memory** (relative to the security parameter). And that's not practical.

Hash Functions in Practice

- In practice we implement a hash function as (an efficient) **pseudo-random-function**



- But wait, we need more!

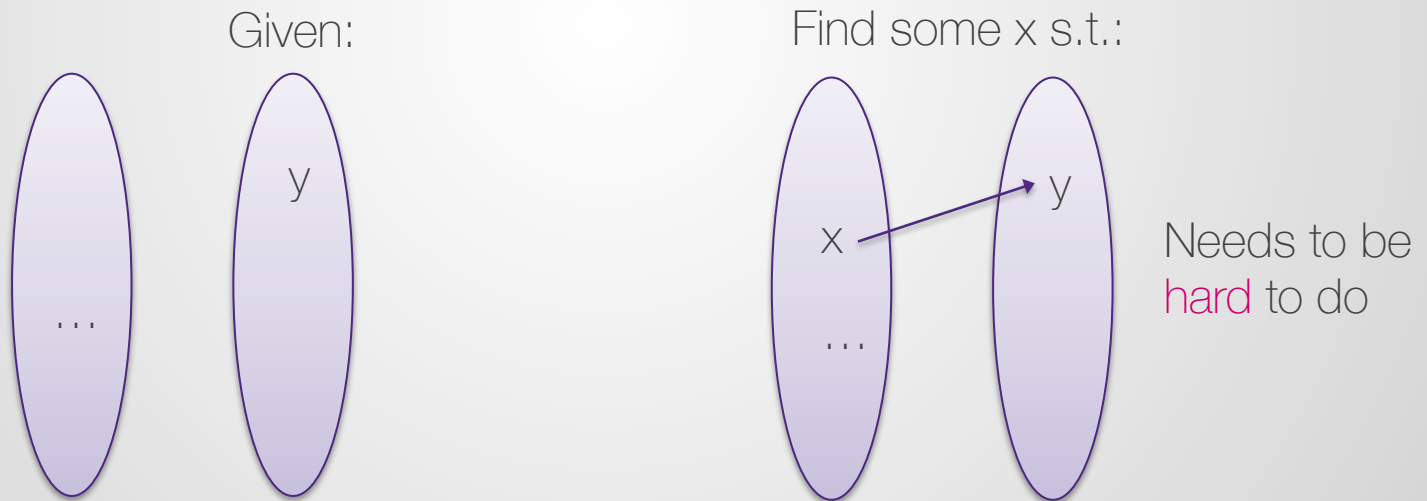
Properties of Cryptographic Hashes

There are three important security properties we need from a hash function:

- Pre-image resistance
 - Given a hash, it should be hard to find a message that produces that hash
- Second pre-image resistance
 - Given a message, it should be hard to find *another* message that produces the same hash
- Collision resistance
 - It should be hard to find any two messages that give the same hash (a so-call **collision**)

Pre-image Resistance

- Given $y \in \{0, 1\}^\ell$ (i.e., the output of a hash) it should be computationally infeasible to find an $x \in \{0, 1\}^*$ such that $y = h(x)$



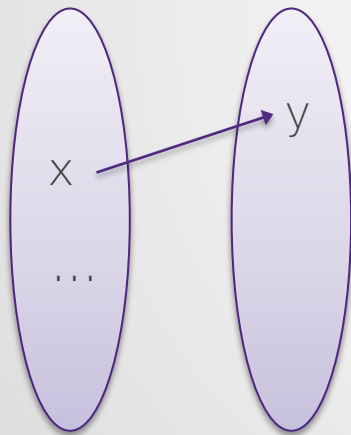
Pre-image Resistance

- If $h(\cdot)$ is a RO, what is the probability of finding a pre-image?
 - Balls in bins analogy: pick an x (i.e., a ball) and throw it into random bin
 - 2^l bins means $1/2^l$ chance of hitting target bin
 - Throw n balls, probability of success grows
 - You can expect to hit the target bin after 2^{l-1} tosses

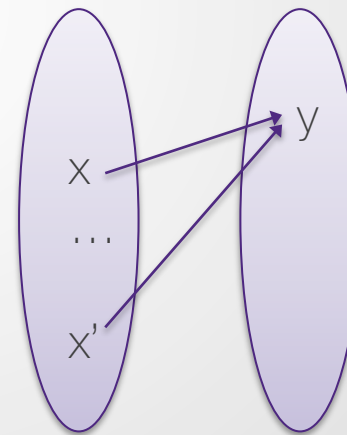
Second Pre-Image Resistance

- Given a pre-image x , find a second *distinct* pre-image x' such that $h(x)=h(x')$

Given:



Find some $x' \neq x$ s.t.:



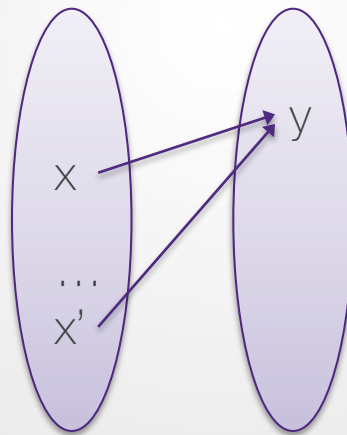
Needs to be
hard to do

- What's the probability here?

Collision Resistance

- It should be computationally infeasible to find *any distinct* pair x, x' such that $h(x)=h(x')$

Find any $x' \neq x$ s.t.:



Needs to be
hard to do

- What's the probability of this?

Birthday Paradox

- A related question: what's the probability there are two people in this room who share the same birthday?
 - If $n=23$, the probability is $\sim 50\%$
 - Number is lower than you might expect, hence "paradox"
- How to compute?
 - Throw two balls. What's the probability they land in same bin (i.e., collide)? $P(\text{single collision})$
 - How many collisions would you expect?
 - $\binom{n}{2} * P(\text{single collision})$
 - Intuition: individual collision probability may be low, but number of possible pairings is high
 - How do these two values grow wrt to each other?

Collision Resistance

- You can expect to find a collision after $2^{\ell/2}$ attempts
- Implication for hash length: if 2^f is considered an infeasible number of operations, then we need:

$$2^f \leq 2^{\frac{\ell}{2}}$$

$$f \leq \frac{\ell}{2}$$

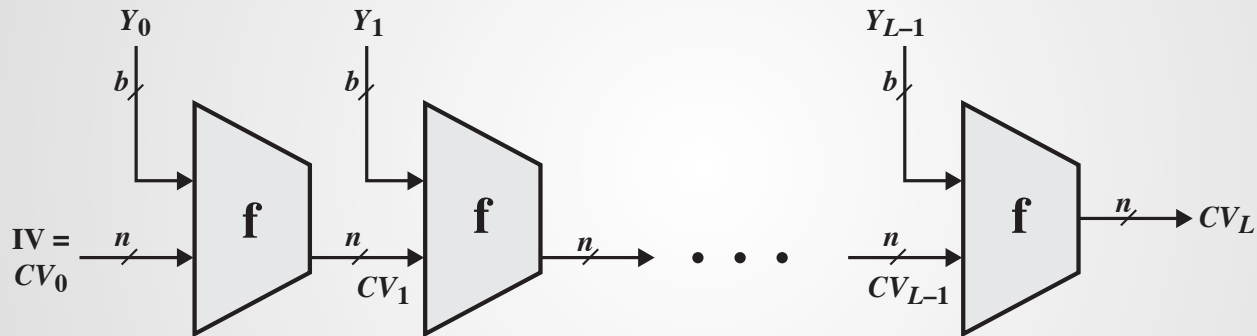
$$2f \leq \ell$$

- So if 2^{128} is considered infeasible, what should the hash length be to resist collisions?

SHA Hash Family

Merkle-Damgård Construction

- F is called a “compression function”



IV = Initial value
 CV_i = chaining variable
 Y_i = i th input block
 f = compression algorithm

L = number of input blocks
 n = length of hash code
 b = length of input block

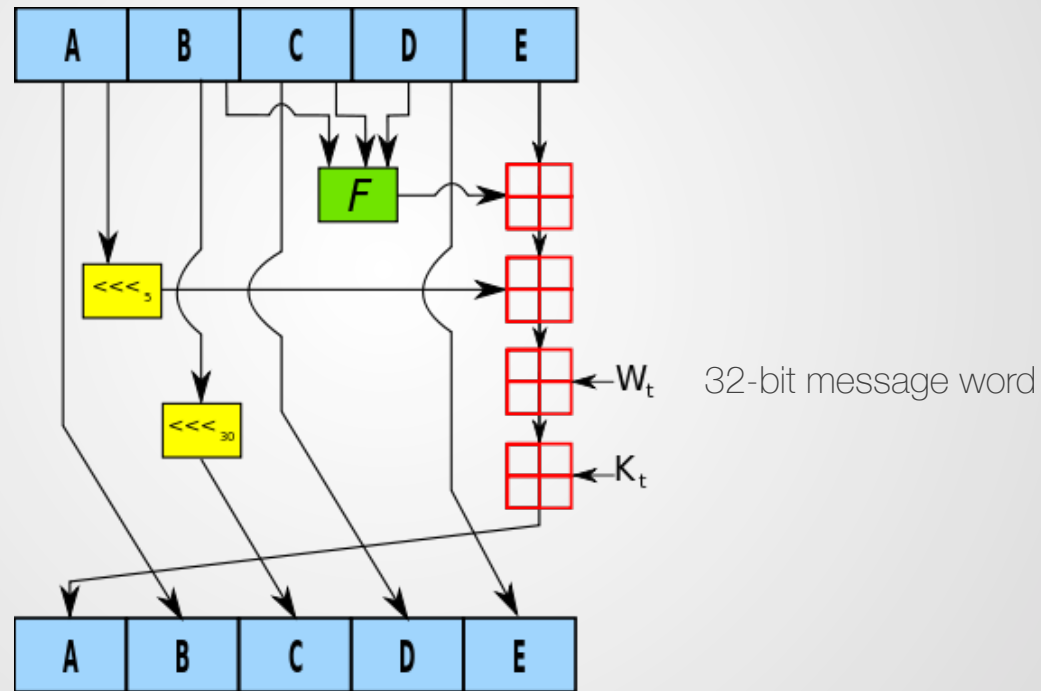
Figure 11.8 General Structure of Secure Hash Code

See course text (Stallings)

SHA Family

- SHA: Secure Hash Algorithm
- NIST standardized in 1993/93
- SHA-1 produces 160-bit output
- 2002 added 256- 384- and 512- bit variants

SHA-1 Compression Function



<http://en.wikipedia.org/wiki/SHA-1>

Repeated 80 times

Other hashes

- MD5
 - Length = 128-bits
 - Collisions in 2^{64} work (naïve, but computationally feasible in 2014)
 - Flaws discovered that can generate collisions much sooner
 - Might make a good assignment question
 - Pre-image attack in 2^{123} work
 - People still use it for some reason. It's creator Ron Rivest encourages you don't anymore.
- Keccak (SHA-3)
 - Sponge construction (very neat alternative to Merkle-Damgård)
 - Upcoming NIST standard

Other issues

- Length extension attacks